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*Gyong Wal Jang, Nam Chol Yu, Hyon Chol Hwang, Guan Sik Jang & Tae Jin Bang*

*Kim Chaek University of Technology*

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*Classification:* FOR Code: 861499

*Language:* English



LJP Copyright ID: 392964  
Print ISSN: 2631-8474  
Online ISSN: 2631-8482

London Journal of Engineering Research

Volume 21 | Issue 1 | Compilation 1.0



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# Calculating Method of Tool Tilt Angle by Solving of Quadratic Equations in 5-Axis Machining with Flat-End Cutter

Gyong Wal Jang<sup>α</sup>, Nam Chol Yu<sup>σ</sup>, Hyon Chol Hwang<sup>ρ</sup>, Guan Sik Jang<sup>ω</sup> & Tae Jin Bang<sup>✉</sup>

## ABSTRACT

*This paper proposes a method for calculating the tilt angle of the tool and the machining strip width when the tool is inclined to the feed direction. We considered the geometry of the surface and the tool as well as the scallop height to determine the tool tilt angle, thus ensuring the tool to be contacted with the surface at two points. This allows us to calculate the tool tilt angle and the machining strip width by solving quadric equations based on the contact circle. An experiment of machining on the quintic B-spline surface was performed and the results show that the proposed method has considerably higher machining efficiency than the CMM.*

**Keywords:** flat-end cutter; tilt angle; machining strip width; scallop height.

**Author α ✉:** Robotics Institute, Kim Chaek University of Technology, Pyongyang, DPR Korea.

**σ:** School of Science and Engineering, Kim Chaek University of Technology, Pyongyang, DPR Korea.

**ρ:** Control Machine Institute, Academy of state science, Pyongyang, DPR Korea.

**ω:** Faculty of Mechanical Science and Technology, Kim Chaek University of Technology, Pyongyang, DPR Korea.

## 1. INTRODUCTION

5-axis machining tools are widely used for sculptured object machining in aircrafts, automobiles, molds, dies, etc. A sculptured surface is often machined with a ball-end cutter in a 3-axis NC machine tool. Traditionally, the ball-end cutter is widely used in sculptured surface machining for the highly flexible controllability. But the process efficiency is low especially for

freeform surface machining, which generally needs multiple tool-paths. Some researchers describe that the feasible adjustment of the tool orientation by the two additional degrees of freedom in flat-end or filleted-end cutter 5-axis machining achieves higher efficiency than ball-end cutter 3-axis machining [1].

The main problems in 5-axis tool path generation are feasible tool orientation determination, avoidance of tool collisions, tool path phase, etc., of which the determination of tool orientations is the most important in increasing the machining strip width within a given tolerance. In 5-axis NC machining, flat-end or filleted-end cutters are known to be much more effective than a ball-end cutter with same diameter. Therefore, many methods have been proposed to determine the tool orientation with different types of cutters. To improve the machining efficiency of multi-axis flank milling of freeform surface, Ming Luo et al proposed a novel barrel cutter design method [2].

The simplest Sturz method has been widely used in the commercial CAD/CAM systems such as UGS NX. In the Sturz method, the tool is traditionally inclined for a constant angle that may range from 5 to 15° to the feed direction about the CC point in the plane containing the CC point, the feed direction and the surface normal. If the tilt angle is too high, then lower gains in metal removal rate will occur. Otherwise, if it is too low, then the tool may gouge the designed surface. Therefore, the tilt angle must be chosen carefully so that no rear gauging occurs during the machining. But, it could not be expected that the machining would be efficient because the tilt angle is determined irrespective of the geometry of the tool and the surface in the neighborhood of

the contact point. A searching method in the machining configuration space (C-space) is proposed to find the optimal tool orientation by considering the local gouging, rear gouging and global tool collision in machining. By using the minimum cusp height as the objective function, first determine the locally optimal tool orientation in the C-space to minimize the machined surface error. Considering the adjacent part geometry and the alternative feasible tool orientations in the C-space, tool orientation are then globally optimized and smoothed to minimize the dramatic change of tool orientation during machining [3]. Tool postures of the flat-end cutter can make a huge difference to both machining strip width and machining efficiency in five-axis end milling. Most of current methods evaluate the machining strip width and implement a tool orientation optimization by finding two intersection points between the effective cutting profile of a flat-end cutter and the offset surface profile which represents machined surface. However, real machining strip width and real residual height should be formed between two adjacent cutter contours. In order to solve the above problems, Baohai Wu et al presented and proved a more suitable method for computing machining strip width [4]. Rao et al. examined the effect of feed on the machined surface for tool orientations using the Principal Axis Method (PAM). In the PAM, the orientation of tool is determined based on the geometric properties of tool and surface at the CC point. The feed direction is selected to be in the direction of minimum curvature calculated at each CC point [5]. The PAM is a special case for the curvature matched machining (CMM). The iterative search for an angle that does not cause gouging used the tilt angle calculated from the maximum curvature as a starting point. Each tool orientation must be incrementally checked for gouging and adjusted until it is gouge-free. It is known that PAM provides lower efficiency than MPM. Lee proposed a non-isoparametric tool path generation method, which feasibly evaluates the machining strip width in flat-end cutter 5-axis surface machining. In this method, the disjointed segments of non-parametric offset path should be connected for smooth tool movements. Furthermore, discontinuities in the surface curvature may

also cause lacking smoothness in the trajectory of a tool center or in the curve of cutter contact points on a surface. The fixed tilt angle of tool was used in his experiments [6].

Generation of efficient tool paths is essential for the cost-effective machining of parts with complex free-form surfaces. A new method to generate constant scallop height paths for the efficient five-axis machining of free-form surfaces using flat-end mills is presented. The tool orientations along the tool paths are optimized to maximize material removal and avoid local gouging. The distances between adjacent tool paths are further optimized according to the specified scallop height constraint to maximize machining efficiency. The constant scallop height tool paths are generated successively across the design surface from the immediate previous tool path and its corresponding scallop curve. An offset surface of the three-dimensional design surface based on the specified scallop height, is used to establish accurately the scallop curve with the constant scallop height [7]. Lo presented a method of tuning adaptively the tilt angle of the flat-end cutter so that the machining strip width can be as large as possible in 5-axis machining with a flat-end cutter. In this method, the cutter paths were scheduled so that the scallop height formed by two adjacent machining paths was constant [8]. Anotaipaboon et al. presented the concept of an adaptive space-filling curve for tool path planning for five-axis NC machining of sculptured surfaces. Generation of the adaptive space-filling curves requires three steps: grid construction, generation of the adaptive space-filling curve, and tool path correction. The space-filling curves, adapted to the local optimal cutting direction, produce sculptor tool paths. This method is endowed with a new modification of techniques for computing the machining strip width along with a modified formula for the minimum tool inclination angle to avoid gouging [9]. Chiou et al. presented a machining potential field (MPF) method to generate tool paths for multi-axis sculptured surface machining. A machining potential field is constructed by considering both the part geometry and the cutter geometry to represent the machining-oriented information on

the part surface for machining planning. The largest feasible machining strip width and optimal cutting direction at a surface point can be found on the constructed machining potential field. The tool paths can be generated by following the optimal cutting direction. Feasible cutter sizes and cutter orientations can also be determined by using the MPF method [10].

Some researchers proposed a Coordinate Matched Method (CMM) that the effective cutting radius of the tool matches the curvature radius of the section curve on the plane normal to the surface at the CC point for better machining efficiency in 5-axis surface machining. Wang et al. presented a 3D curvature matched machining method and a curvature gouge detection and elimination method for 5-axis surface machining. This method is based on the Euler-Meusnier spheres concept and the geometric model of surface curvature geometry [11]. Shanming Luo et al. proposed the curvature matching method and the minimum distance method to inspect interferences occurring in the five-axis end milling of cycloidal gears [12]. Jensen et al. presented a method for determining the tool orientation based on the CMM in 5-axis finish surface machining. The tool is inclined to the direction corresponding to minimum principle curvature of the designed surface, and the tilt angle is adjusted until the “effective cutting radius” of tool is equal to the radius of maximum curvature of the designed surface at the CC point [13].

Some researchers presented various methods for determining the optimal tool orientation.

Wang et al. presented a 5-axis tool-path generation algorithm for alleviating the abrupt change of tool orientations when the tool orientation was adaptively determined to avoid the local and global gouge [1]. Feng et al. examined the effect of tool tilt angle on the machining strip width in determining the optimal tool orientation and feed direction in 5-axis flat-end milling [14].

On the other hand, Warkentin et al. proposed a Multi-Point Machining (MPM) method for higher effectiveness in 5-axis NC machining. In MPM,

there exist more than one-point contact between the desired surface and the cutting tool [15]. Also proposed an efficient algorithm that generate the multi-point positions of tool. Their algorithm was found to be less effective for complex surface machining [15] and [16]. Warkentin et al. compared MPM with the Sturz method and the PAM, and showed that the scallop height generated in 5-axis machining using MPM is much smaller than the others [17]. In MPM, the tool is positioned such that there are two CC points. This method is mathematically complicated because it needs search of specific areas for potential candidates for the second cutter point for each tool position after the first CC point is predetermined. The tool is forced to maintain contact at the first CC point. It is then rotated about two independent axes until an optimal position is achieved that minimizes the distance between the tool and the second CC point. Among two independent axes, one is the feed direction and the other is a vector normal to the plane that includes the feed direction and perpendicular to the surface. This method is still much complicated than other methods and would have a limited use in industry due to the complexities of the algorithm.

Together with the tool orientation, the tool path phase has a large influence on the machining efficiency. The simplest approach is the iso-parametric method which use the curves of constant parameter. For the iso-parameter machining, if the feed direction is set along the  $u$  parameter, the tool paths are generated by incrementing the  $v$  parameter. Here, the path interval must be set carefully because the scallop height between the tool paths could not be constant. Another approach for the tool path generation is to use intersection curves of parametric surface and a series of vertical planes. For this iso-plane machining, the path interval or the distance between the vertical planes is also determined based on the scallop height limitation. For both iso-parameter and iso-plane approaches, the scallop height could not be maintained to be constant. The iso-scallop height approach was proposed to overcome this drawback.

The Point-In-Convex-Hull-Control (PICHC) method is presented for the calculation of the optimum tool orientation during the 5-axis simultaneous CNC machining of sculptured surfaces. More specifically, this novel exact multi-point algorithm minimizes the clearance and/or overclosure of the contact surface between the tool and the work piece and maximizes the material removal rate [18]. Convex edge surfaces are always encountered in industry, such as the leading and trailing edges of turbine blades. Concave cutter can be used to flank machine such surfaces. The fundamental theory is given by investigating the conjugation between a concave cutter and a cylindrical surface. By analyzing the projection of the grazing curve in the cross-section plane, it finds that convex edge surface with various cross-section radiuses can be machined with a concave cutter by adjusting the cutter orientation [19]. Machchhara et al. presents an algebraic based approach and a computational framework for the simulation of multi-axis CNC machining of general freeform tools. The boundary of the swept volume of the tool is precisely modeled by a system of algebraic constraints, using B-spline basis functions [20]. The Drop and Spin Method (DSM), a multipoint tool positioning technique for five-axis machining of corners formed by the intersection of two bi-quadratic Bzier surfaces with a toroidal end mill is presented by Kumar Sharma et al. In the DSM the tool is dropped onto one of the surfaces to identify the first point of contact. The second point of contact is determined on the second surface. These two points of contact ensure that the toroidal tool can be positioned in the closest possible proximity to the two surfaces along the common edge [21]. To solve the problem of the poor machining quality of the leading and trailing edge surface of the aircraft engine blade, a tool path generation method based on smooth machine rotary angle and tilt angle in five-axis surface machining with torus cutters was proposed by Zhang et al. In terms of a specified type of five-axis machine tool, a relationship equation between design variables of tool position and machine rotary angles was firstly derived. A new tool orientation smoothing approach was then put forward. On this foundation, a tool path

generation method based on a smooth machine rotary angle and tilt angle was presented [22].

As mentioned above, a flat-end cutter is still widely used in industry with a fillet-end cutter. Many previous studies considered only a single CC point in flat-end cutter 5-axis machining when determining the tool orientation. The machining efficiency could be increased with multi contact points as MPM. In this paper, we propose a simple method for calculating the tool tilt angle and the machining strip width when the tool is inclined to the feed direction in order to maximize the machining strip width in flat-end cutter five-axis surface machining. And we verify the effectiveness through the machining experiments.

This paper is divided into five sections. The section 1 gives a brief overview of various concepts and notations in 5-axis machining and then, Section 2 gives explanations of the method for calculation of tool tilt angle and machining strip width based on the contact circle and the scallop height. In Section 3, the machining experiments are shown based on the proposed method.

## II. THEORETICAL BACKGROUND

Fig.1 shows a rotated flat-end cutter of radius  $R$  at a **CC** point and the associated coordinate systems in 5-axis surface machining with a flat-end cutter. A local coordinate system is established at the **CC** point on the design surface. The **CC** point is the origin of this local coordinate system. In the local coordinate system,  $\mathbf{n}$  is the unit vector in the surface normal direction and  $\mathbf{f}$  is the unit vector in the feed direction and  $\mathbf{k} = \mathbf{n} \times \mathbf{f}$ .

In 5-axis machining, the tool orientation is defined by 2 rotation angles. The tool is first rotated about  $\mathbf{t}$  for an tilt angle  $\phi$  and then about  $\mathbf{n}$  for a tilt angle  $\beta$ . Tool coordinate system (TCS) is established by the center of the bottom tool circle (BTC) of the flat-end cutter as the origin and tool axis as  $\mathbf{t}_{\text{axis}}$ .

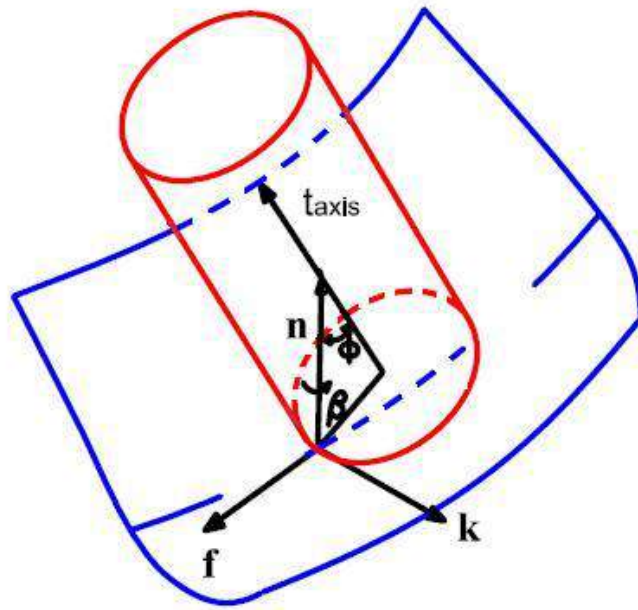


Fig. 1: Machining with flat-end cutter

It was known that the machining efficiency is good when the feed direction is chosen as the maximum curvature direction [7]. However, they assume that the feed direction is arbitrarily predefined.

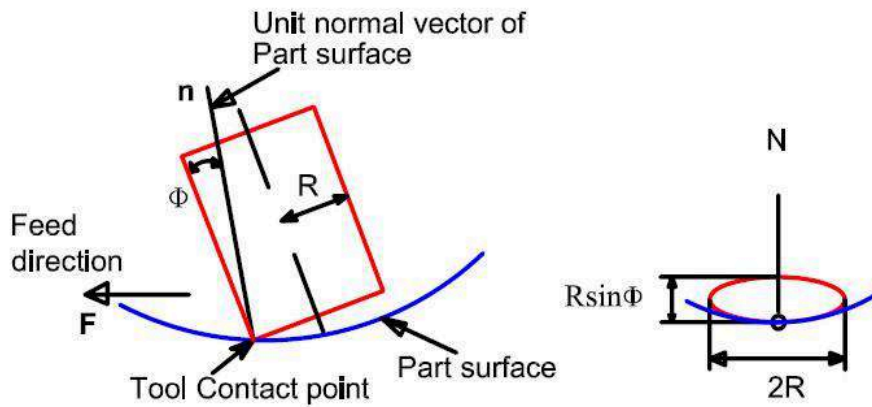


Fig. 2: Cutting ellipse

When the contact point between the tool and the designed surface is only one, the orthogonal projection of the bottom edge of tool on k-n plane is called the cutting ellipse and the curvature radius of this ellipse at CC point is called the cutting radius as shown in Fig. 2. In Fig. 3, the tool axis is inclined for an angle  $\phi$  to the feed direction and contacts with the part surface at CC point. The plane formed by the tool axis and the feed direction is called the tool axis plane.

A unit vector  $e$  is defined as:

$$e = n \times (n \times f)$$

Then the unit vector for the tool axis is

$$t_{axis} = \cos(\phi)n + \sin(\phi)e \quad (1)$$

And the tool position  $t_{pos}$  can be expressed as

$$t_{pos} = cc + R\sin(\phi)n - R\cos(\phi)e \quad (2)$$

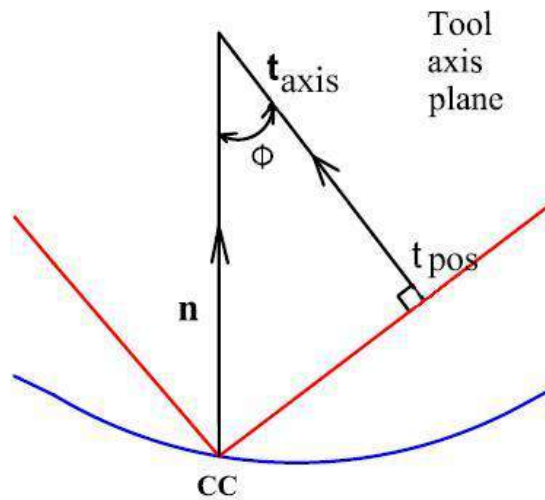


Fig. 3: Situation of tool in the tool axis plane

Suppose that the local normal curvature of the machined surface at the CC point perpendicular to the feed direction is  $\kappa$ . Then, the cutting radius is

$$\frac{\sin(\phi)}{R}$$

when the tool tilt angle is  $\phi$ . Thus,

$$\kappa = \frac{\sin(\phi)}{R}$$

As a result, the tool tilt angle can be determined by using the local normal curvature of the surface as follows:

$$\phi = \arcsin(\kappa \cdot R) \quad (3)$$

Calculation of a machining strip width by one contact point method was carefully explained in [6].

### III. METHOD

#### 3.1: Determination of a tool tilt angle based on the contact circle

The scallop height forms the residual after machining and is removed during the fine finish process. Previously, the machining strip width was evaluated based on this scallop height. The scallop height is an important factor that effects on the machining strip width. The machining strip was defined as the machined region that lies within the required scallop height limits, where

the scallop height was not greater than the tolerance. From the viewpoint of the permissible tolerance, this means that the scallop height is not greater than the permissible tolerance at all points within the machining strip, and never means that the tool must be surely contacted with the surface within the machining strip.

Now, we consider the tool orientation in which the tool contacted with the machined surface at 2 points and with the offset surface at one point as shown in Fig. 4. The tool is assumed to be inclined to a predetermined direction. In Fig. 4, the point  $cc_0$  is generated by translating the reference contact point  $cc$  with the given tolerance  $\delta$  to the direction perpendicular to the surface. The tool is inclined to the machining direction about the axis  $k_i$ , which passes through the point  $cc_0$  and perpendicular to the machining direction.

The intersection curve of the  $t$ - $n$  plane and the machined surface can be approximated by a contact circle with a radius equal to the major radius of the effective cutting ellipse. Here, the bottom circle of the inclined tool intersects at two points with the contact circle when the tool is inclined in the feed direction as shown in Fig. 4. Fig. 4 also shows the situation of the cutting ellipses in the case of one contact point. As shown in Fig. 4, we can expect that the machining strip width  $W$  would be much greater than the

machining strip width  $W_{FAM}$  at only one contact point.

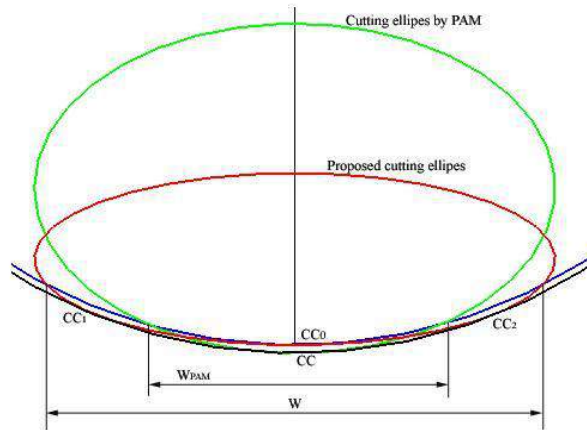


Fig. 4: Arrangement of cutting ellipses at one point of contact and two points of contact

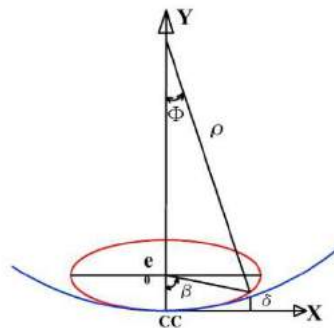


Fig. 5: Relation between the contact circle and the cutting ellipse at a contact point

First, we consider the case in which the cutting ellipse contacts with the contact circle at the cc point. In Fig. 5,  $\rho$  is the radius of the contact circle. As shown in Fig. 5, the points on the circumference of the contact circle in the coordinate system with the cc point as the origin is described as follows:

$$\begin{cases} x = \rho \sin \phi \\ y = \rho - \rho \cos \phi \end{cases}$$

Then

$$\sin \phi = \frac{x}{\rho}, \cos \phi = \frac{\rho - y}{\rho}$$

Therefore,

$$\frac{x^2}{\rho^2} + \frac{(\rho - y)^2}{\rho^2} = 1$$

$$x^2 = 2\rho \cdot y - y^2 \quad (4)$$

Fig. 6 shows the situation of the cutting ellipses in the case of two contact points. Let's consider the points on the circumference of the cutting ellipse, which are the points on the bottom edge of the tool.

The tool is inclined about  $k_1$  until the tool contacts with the contact circle at two points.

When the tool is inclined as long as  $\phi$  in the feed direction, the cutting ellipse has the major radius  $a=R$  and the minor radius  $b=R \sin \phi$ . Then, the center of the ellipse is represented as:

$$\begin{cases} x = 0 \\ y = \delta + R \sin \phi \end{cases}$$

The points on the circumference of the ellipse are represented as:

$$\begin{cases} x = a \cos \beta = R \cos \beta \\ y = \delta + R \sin \phi - R \sin \phi \sin \beta \end{cases}$$

Thus,

Thus,

$$\cos \beta = \frac{x}{R}, \quad \sin \beta = \frac{\delta + R \sin \phi - y}{R \sin \phi} \frac{x^2}{R^2} + \frac{(\delta + R \sin \phi - y)^2}{R^2 \sin^2 \phi} = 1 \quad (5)$$

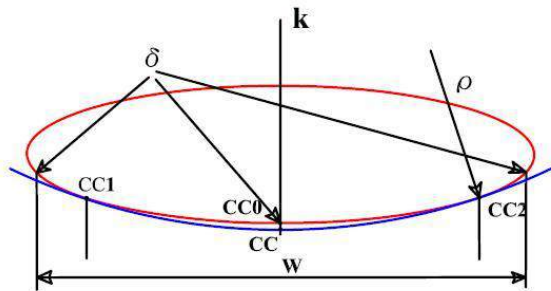


Fig. 6: Relation between the contact circle and the effective cutting ellipse at two contact points.

From Eqs. (4) and (5), we can obtain the position

$(x(\phi), y(\phi))$  of the point at which the cutting ellipse and the contact circle are contacted.

$$2\rho \cdot y - y^2 = R^2 - \frac{1}{\sin^2 \phi} (\delta + R \sin \phi - y)^2 \quad (6)$$

$$(1 - \sin^2 \phi)y^2 + 2(\rho \sin^2 \phi - \delta - R \sin \phi)y + \delta^2 + 2\delta R \sin \phi - R^2 \sin^2 \phi = 0$$

Now, the determinant for extreme value of  $y$  is as following.

$$D = (\rho \sin^2 \phi - \delta - R \sin \phi)^2 - (1 - \sin^2 \phi)(\delta^2 + 2\delta R \sin \phi - R^2 \sin^2 \phi) = 0$$

Let's  $\sin \phi$  be  $X$ . Then the above equation can be rewritten as:

$$(\rho X^2 - \delta - RX)^2 - (1 - X^2)(\delta^2 + 2\delta RX - R^2 X^2) = 0 \quad (\rho^2 - R^2)X^4 + (-2\rho\delta R + 2\delta R)X^3 + (-2\rho\delta + 2R^2 + \delta^2)X^2 = 0$$

$$(\rho^2 - R^2)X^2 - 2\delta R(\rho - 1)X + (2R^2 - 2\rho\delta + \delta^2) = 0$$

where the solution  $X = \sin \phi = 0$  is ignored.

$$X_{1,2} = \frac{\delta R(\rho - 1) \pm \sqrt{(\delta R(\rho - 1))^2 - (\rho^2 - R^2)(2R^2 - 2\rho\delta + \delta^2)}}{(\rho^2 - R^2)}$$

Therefore, the tilt angle of the tool is calculated as:

$$\phi_{1,2} = \arcsin X_{1,2} \quad (7)$$

From the two solutions, the minimum value is only selected because the smaller the tilt angle, the larger the machining strip width.

equation for the contact circle and the value  $y$  of the cutting ellipse is equal to  $\delta$ . As a result, instead of Eq. (6), we can get the following equation.

The machining strip width can be calculated from the fact that the difference between the value  $y$  of

$$2\rho \cdot (y + \delta) - (y + \delta)^2 = R^2 - \frac{1}{\sin^2 \phi} (\delta + R \sin \phi - y)^2 \quad (8)$$

$$2\rho \cdot (y + \delta) \sin^2 \phi - (y + \delta)^2 \sin^2 \phi = R^2 \sin^2 \phi - (\delta - y)^2 - 2(\delta - y)R \sin \phi - R^2 \sin^2 \phi^2$$

$$(1 - \sin^2 \phi)y^2 + 2(\rho \sin^2 \phi - \delta \sin \phi - R \sin \phi)y + 2\rho \delta \sin^2 \phi + \delta^2 + 2\delta R \sin \phi - R^2 \sin^2 \phi - \delta^2 \sin^2 \phi = 0$$

From Eq. (8), two values  $y_{1,2}$  can be calculated. From the positive values, we select the greater value, and twice that value must be not greater than  $2R$ . Then, the machining strip width  $W$  is calculated as:

$$W = 2y \quad (9)$$

Now, the tool tilt angle and the machining strip width along with it can be easily calculated by using analytic methods.

In this way, we determined the tool orientation where the machining strip width could be increased as much as possible when the tool was inclined in the predefined direction, thus establishing the calculating method for the machining strip width.

If  $\kappa_1$  and  $\kappa_2$  are the principal curvature of the designed surface at the reference contact point  $\mathbf{cc}$ ,  $\rho$  becomes the maximum principal curvature radius corresponding to  $\kappa_1$  when the predetermined feed direction is selected as the minimum principal curvature direction. Then the tool tilt angle and the machining strip width can be also calculated by Eqs. (7), (8) and (9).

Let the angle between the feed direction and the direction of the minimum curvature be denoted as  $\theta$ , then the machining strip width is described as:

$$W_1 = W \cdot \cos(\theta) \quad (10)$$

When  $\theta = 0^\circ$ , i.e. the machining direction coincides with the direction of the minimum curvature, then the machining strip width is maximum, while when  $\theta = 90^\circ$ , it is minimum.

Feng et al. [10] studied the influence of the tilt angle on the machining strip width and showed that the direction of minimum curvature is not always the best feed direction for machining all the area of the surface. However, that is still a relatively good direction. If the tilt angle is not so large, the machining strip width in the feed

direction can be approximately evaluated by Eq. (9).

Also, based on the reference contact point CC and the tool tilt angle, we can generate the CL data by using Eqs. (1) and (2).

### 3.2: Tool path search

In 5-axis surface machining, the primary tool path usually starts with the boundary curve of the surface. After determining the primary tool path, the adjacent paths must be determined sequentially.

Lee studied carefully the methods to avoid the tool gouge and obtain the adjacent contact points in 5-axis machining with a flat-end cutter [6].

When the machining strip width,  $W_o$ , is determined at the current reference contact point  $S(u_o, v_o)$  and is assumed to be the distance between the primary and the adjacent tool paths, the adjacent contact point can be calculated from Eq. (11).

$$\frac{\partial \mathbf{S}}{\partial u} \Delta u + \frac{\partial \mathbf{S}}{\partial v} \Delta v = W_o \mathbf{k} \quad (11)$$

When  $\Delta u$  and  $\Delta v$  – the increment values of parameters  $u$  and  $v$ , respectively – are calculated from Eq. (11), the parameters of the reference contact point of the adjacent path are as:

$$u = u_o + \Delta u, \quad v = v_o + \Delta v$$

However, this point  $S(u_o + \Delta u, v_o + \Delta v)$  cannot always be the correct adjacent reference contact point because of the assumption that the machining strip width  $W_o$  is the distance between the primary and the adjacent standard contact points.

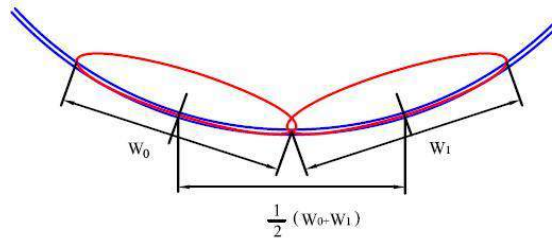


Fig. 7: Search scheme of the adjacent reference contact point

Therefore, regarding  $S(u_0 + \Delta u, v_0 + \Delta v)$  as a reference contact point, the machining strip width must be calculated again. In this case, the candidate feed direction is the straight line that connects the two predetermined adjacent

reference contact points. And the final parameters of reference contact points of the adjacent tool path are determined by recalculating  $\Delta u$  and  $\Delta v$  based on the following equation:

$$\frac{\partial \mathbf{S}}{\partial u} \Delta u + \frac{\partial \mathbf{S}}{\partial v} \Delta v = \frac{1}{2} (W_0 + W_1) \mathbf{k} \quad (12)$$

The local and global gouges in 5-axis machining are solved by the methods proposed by Lee. (1998). And the tool tilt angle and tilt angle are adjusted at  $cc_0$ .

machine tool is a dual tool head rotation type (Fig 6). This surface is thought to be such complicated that the efficiency of the proposed method could be verified and concave. The details of values of the control points of the surface are listed in Table. 1.

#### IV. RESULT AND DISCUSSION

The surface to be machined is a quintic B-spline surface with 36 control points and the CNC

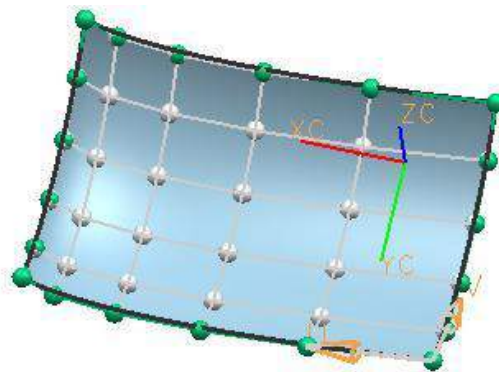


Fig. 8: B-spline surface and control points

The surface is included in the cubic of  $187 \times 124 \times 53$ (mm) and the minimum curvature radius is 28.25mm. The flat-end cutter with  $R = 30$ mm was selected for surface machining.

The allowance for finishing was 0.5mm and the tolerance was 0.03mm. The tool path was generated along with the iso-parametric curves

( $v$ -parameter). The tool path was generated in Zig mode for the observation to the tool paths. Fig. 9 shows the situation of the tool path for machining. The CMM required 29 segments of tool path, while the proposed method required 21. As a result, the machining period is 28percent reduced for machining with the same tolerance.

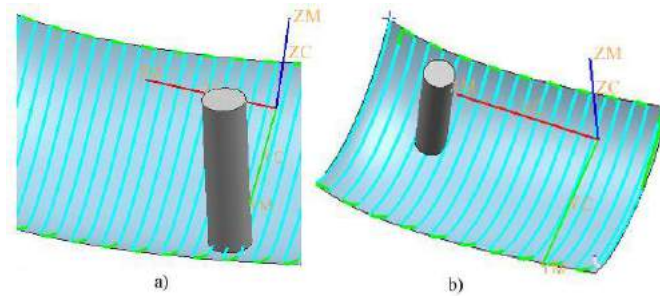


Fig. 9: Tool paths for 5-axis machining by the CMM (a) and the proposed method (b)

Fig. 10 shows the surface deviation produced when machining the sample surface with the CMM and the proposed method. The proposed method can considerably reduce the finishing time because the distributions of residuals are sparse.

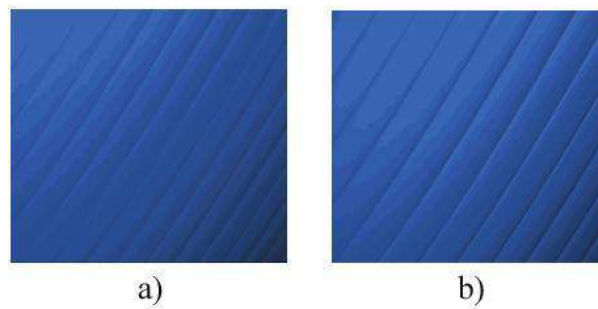


Fig. 10: Surface deviation produced when machining bi-quantic B-spline surface in CMM (a) and the proposed method (b)

Table 1: The control points of sample surface

row		Column1	Column2	Column3	Column4	Column5	Column6
Row1	X	-33.153	24.843	74.154	113.219	140.483	154.39
	Y	88.38	88.38	88.38	88.38	88.38	88.38
	z	0	-15.284	-22.927	-22.927	-15.284	0
Row2	X	-33.153	24.844	74.154	113.219	140.484	154.39
	Y	67.235	67.235	67.235	67.235	67.235	67.235
	z	-19..12	-34.405	-42.047	-42.047	-34.405	-19..12
Row3	X	-33.153	24.843	74.154	113.219	140.484	154.39
	Y	40.568	40.568	40.568	40.568	40.568	40.568
	z	-28.68	-43.965	-51.607	-51.607	-43.965	-28.68
Row4	X	-33.153	24.844	74.154	113.219	140.484	154.39

	Y	12.061	12.061	12.061	12.061	12.061	12.061
	z	-28.68	-43.965	-51.607	-51.607	-43.965	-28.68
Row5	X	-33.153	24.844	74.154	113.219	140.484	154.39
	Y	-14.606	-14.606	-14.606	-14.606	-14.606	-14.606
	z	-19.12	-34.405	-42.047	-42.047	-34.405	-19.12
Row6	X	-33.153	24.843	74.154	113.219	140.484	154.39
	Y	-35.751	-35.751	-35.751	-35.751	-35.751	-35.751
	z	0	-15.284	-22.927	-22.927	-15.284	0

The proposed method is expected to improve concave surface machining. But for the convex surface, the machining efficiency is lowered due to the reduced machining strip width. In this case, one contact point method can be used as in [6].

## V. CONCLUSIONS

This paper proposed a method to calculate the tool tilt angle and the machining strip width based on the predefined scallop height in machining where the tool contacts with the surface at two points.

First, we considered the geometry property of the surface and the tool as well as the scallop height, thus considering the tool to be contacted with the surface and the tool. Second, we also calculated the tool tilt angle and the machining strip width by solving quadric equations based on the contact circle. Third, we experimented the machine of proposed method and CMM on the biquantic B-spline surface and ensured machining efficiency. In the proposed method, the tool tilt angle and the machining strip width could be analytically calculated by solving a quadric equation based on the contact circle. The example have shown that the machining time can be considerably saved and the productivity increased by using the proposed method compared with the CMM.

- Ethical Approval
- Consent to Participate
- Consent to Publish
- Authors Contributions

- Funding
- Competing Interests
- Availability of data and materials

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